Tutorial Note for Math2012E

May 30, 2016

1 Functions of several variables

- domain/range
- level set $\{\vec{x}|f(\vec{x}) = c\}$
- boundary/interior
- open/closed
  - U is open if $\forall x \in U, \exists$ open set V s.t. $x \in V \subset U$
  - U is open if all points are interior points
  - U is open if and only if complement is closed
  - U is cloed if it contains its entire boundary
  - U is cloed if it contains all its limit points.

cf. principles of mathematical analysis, Rudin

Exercises:

- Prove that U is open if and only if complement is closed using other equivalent definitions.
- Prove that all definitions are equivalent

2 limits and continuity

- Definition of limit

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L \text{ if } \forall \epsilon > 0, \exists \delta > 0, \text{ s.t. } d(f(x,y), L) < \epsilon \text{ whenever } d((x,y),(x_0,y_0)) < \delta$$

- properties of limit
  - sum/difference/scalar multiple
  - product/quotient

- power/root

- two-path test for nonexistence of a limit
- Definition of continuity at a point (x_0, y_0)

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$$

- Properties of continuity:
  - preserved under composition of continuous map

Exercise:

- Use math language to give a precise description of the above properties of continuity and then prove it.

3 Partial Derivatives

- Definition of partial derivative : regard other variables as constant and do like 1-variable case
- higher-order differential is independent of order i.e.

$$f_{xy} = f_{yx}$$

- Definition of differentiable: $f(x,y)$ can be written down as following:

$$\triangle z = f_x(x_0,y_0)\triangle x + f_y(x_0,y_0)\triangle y + \epsilon_1\triangle x + \epsilon_2\triangle y$$

- partial differential exist at a point $\nRightarrow$ differentiable at that point
- partial derivatives are continuous in an open region $R \Rightarrow$ differentiable in that region
- partial differential exist $\nRightarrow$ continuity
- differentiability $\Rightarrow$ continuity

Exercises:

- Give an example of partial differential exist at a point $\nRightarrow$ differentiable at that point and show it does satisfy the condition.
- Give an example partial differential exist $\nRightarrow$ continuity annd show it does satisfy the condition.
- Prove that differentiability $\Rightarrow$ continuity using definition.